1. (20 points) Let $I = [-1, 1]$. Let $g : I \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \sin(\frac{1}{x}) & \text{if } x > 0 \\ x & \text{otherwise} \end{cases}$$

Show that g is Riemann Integrable.

2. Let $v \in \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{x_1 x_2^4}{x_1^4 + x_2^6} & \text{if } \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) (10 points) Let $v \in \mathbb{R}^2$ be non-zero. Find the directional derivative in the direction v of g at 0.

(b) (10 points)Is g differentiable at 0 ?

3. Decide whether each of the following statements are true or false providing adequate justification.

(a) (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Suppose $f(\mathbb{R})$ is countable then f is a constant function.

(b) (10 points) Let $g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{x_1^3}{x_1^4 + x_2^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then g is continuous at 0.

(c) (10 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a differentiable function. If $\frac{\partial f}{\partial x_1}(0) < 0$ and $\frac{\partial f}{\partial x_2}(0) < 0$. Then $D_u f(0) < 0$ for all $u \in \mathbb{R}^2$.

(d) (10 points) Let $f : [0, 1] \to \mathbb{R}$ be bounded. Then f is Reimann integrable.

4. Let $\mathbb{R}^2$ be a metric space with the euclidean metric. Let $D = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 1\}$

(a) (5 points) Show that D is a compact set in $\mathbb{R}^2$.

(b) (15 points) Let $f : D \to \mathbb{R}$ be given by $f(x) = x_1 + 2x_2^2 + 1$. Find the absolute minimum and maximum of f on D.